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Web Appendix of

Measuring Discounting without Measuring Utility

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Web Appendix WA: Further details for the DM

WA1. Details of rounding in the DM method

As is common with choice lists today, subjects were not given the possibility to express indifference. The latter possibility is known to generate confusions and to be hard to incentivize. Preferences $\alpha_{\{1,\ldots,24\}}0 \prec \alpha_{\{25,\ldots,52\}}0$ and $\alpha_{\{1,\ldots,25\}}0 \succ \alpha_{\{26,\ldots,52\}}0$ reveal that $c_{½}$ is in the interval (24, 25]. We denote upper bounds of such intervals by u, so that here we write $u_{½} = 25$. As is common, we will use midpoints as estimates. Thus $u_{½}$–½ is our estimate of $c_{½}$, which is 24.5 in the above case.

In our adaptive experiment, we only presented integer week durations to subjects, and no noninteger such as $u_{½}$–½. Hence we could not use our best approximation $u_{½}$–½ of $c_{½}$ in follow-up questions, but had to use an integer approximation. So as to stay away from extreme values we used the convention of rounding values upward for time durations in the first half year and downward for time durations in the second half year. Those rounded integer values were presented to our subjects in the adaptive experiment. We obviously correct for the generated and propagated rounding errors in our analyses. We next give details.

We follow the notation in the paper and denote time intervals as (0,x] or, equivalently, as a set {1,…,x} of weeks. Similarly, (x, 52] denotes {x+1,…,52}. Note that week x+1 runs from time point x until time point x+1, so that its left starting point is x and not x+1.

For estimating $c_{½}$ we measured the subjective midpoint of (0, 52] = {1,…,52}. The preference switch (and subjective interval-midpoint) was between $u_{½}$–1 and $u_{½}$, referring to the notation of $u_{½}$ introduced above. Thus, for each subject $u_{½}$ was such that

$(0, u_{½}-1] = \{1,\ldots,u_{½}-1\} \prec \{u_{½}, \ldots, 52\} = (u_{½}-1, 52]$

and

$(0, u_{½}] = \{1,\ldots,u_{½}\} \succ \{u_{½}+1, \ldots, 52\} = (u_{½}, 52]$.

We then assume

$(0, u_{½}-½] \sim (u_{½}-½, 52]$ and, hence, estimate

$c_{½} = u_{½}-½$.

To estimate $c_{¼}$, we measured the preference-midpoint of $(0, u_{½}]$ (weeks $\{1,\ldots,u_{½}\}$), denoted $u_{¼}-½$, similarly as above. There is a propagation of roundings here, as follows. $u_{½}$ overestimates $c_{½}$ by ½ (on average, as always) as we saw, implying that the midpoint of $(0, u_{½}]$ will overestimate $c_{¼}$ by ¼. Hence we subtract ¼ from the subjective midpoint $u_{¼}-½$, and estimate

$c_{¼} = u_{¼}-¾$.

To estimate $c_{⅛}$, we measured the preference-midpoint of $(0, u_{¼}]$ (weeks $\{1,\ldots,u_{¼}\}$), denoted $u_{⅛}-½$. Because $u_{¼}$ overestimates $c_{¼}$ by ¾ as we saw, the subjective midpoint of $(0, u_{¼}]$ will overestimate $c_{⅛}$ by ⅜. Hence we estimate

$c_{⅛} = u_{⅛} - ⅞$.

To estimate $c_{¾}$, we measured the preference midpoint of $(u_{½}-1, 52]$ (weeks $\{u_{½},\ldots,52\}$), denoted $u_{¾}-½$. Because $u_{½}-1$ underestimates $c_{½}$ by ½, the preference midpoint underestimates $c_{¾}$ by ¼, which is to be added to $u_{¾}-½$. Hence we estimate

$c_{¾} = u_{¾} - ¼$.

To estimate $c_{⅞}$, we measured the preference midpoint of $(u_{¾}-1, 52]$ (weeks $\{u_{¾},\ldots,52\}$), denoted $u_{⅞}-½$. Because $u_{¾}-1$ underestimates $c_{¾}$ by ¾, the preference midpoint underestimates $c_{⅞}$ by ⅜, which is to be added to $u_{⅞}-½$. Hence we estimate

$c_{⅞} = u_{⅞} - ⅛$.

For the first separability test, we obtained an estimate $s_{½}$ of $c_{½}$ alternative to the one obtained before. We now measured the preference midpoint of $(u_{¼}-1, u_{¾}]$ (weeks $\{u_{¼},...,u_{¾}\}$), denoted $s^1_{½}-½$. Because $u_{¼}-1$ underestimates $c_{¼}$ by ¼, and $u_{¾}$ overestimates $c_{¾}$ by ¼, the preference midpoint is a good estimate of $c_{½}$. That is, we estimated

$s_{½}$ (alternative for $c_{½}$) $= s^1_{½} - ½$.

For the second separability test, we did not measure a subjective midpoint of a time interval. We obtained an alternative measurement $s_{¾}$ of $c_{¾}$, as follows. The basic idea is to find $s_{¾}$ such that $(0,u_{¼}] \sim (s_{¾},52]$. Roundings are as follows. We found the value $s^2_{¾}$ such that

$\{s^2_{¾}+1, \ldots,52\} = (s^2_{¾},52] \prec (0,u_{¼}] = \{1,\ldots,u_{¼}\} \prec \{s^2_{¾},\ldots,52\} = (s^2_{¾}-1,52]$. We estimate

$(s^2_{¾}-½,52] \sim (0,u_{¼}]$.

Because $u_{¼}$ overestimates $c_{¼}$ by ¾, $s^2_{¾}-½$ will underestimate $c_{¾}$ by ¾. We thus estimate

$s_{¾}$ (alternative for $c_{¾}$) $= s^2_{¾} + ¼$.

WA2. Details of qualitative preference conditions for the DM

We first give the p-values obtained.

TEST 1: H_0: $c_{½} \geq 26$ (no or negative impatience) is rejected to the favor of H_1: $c_{½} < 26$ (impatience; $p < 0.001$).

TEST 2: H_0: $c_{¼} \geq c_{½}/2$ (no or negative impatience) is rejected to the favor of H_1: $c_{¼} < c_{½}/2$ (impatience; $p < 0.001$).

TEST 3: H_0: $c_{½} \geq (c_{¼} + c_{¾})/2$ (no or negative impatience) is marginally rejected to the favor of H_1: $c_{½} < (c_{¼} + c_{¾})/2$ (impatience; $p = 0.09$).

TEST 4: H_0: $c_{¾} \geq (c_{½} + 52)/2$ (no or negative impatience) is rejected to the favor of H_1: $c_{¾} < (c_{½} + 52)/2$ (impatience; $p < 0.001$).

TEST 5. H_0: $c_{⅛} \geq c_{¼}/2$ (no or negative impatience) is rejected to the favor of $c_{⅛} < c_{¼}/2$ (impatience; $p = 0.001$).

TEST 6. H_0: $c_{⅞} < (52 + c_{¾})/2$ (no or negative impatience) is rejected to the favor of $c_{⅞} < (52 + c_{¾})/2$ (impatience; $p < 0.001$).

TEST 7. We tested constant impatience by comparing impatience in $(0,c_{½}]$ versus $(c_{½},52]$, testing $c_{½}/2 - c_{¼} = (c_{½}+52)/2 - c_{¾}$ two-sided. We found constant impatience rejected to the favor of decreasing impatience (with > instead of =; $p < 0.001$).

TEST 8. We tested constant impatience by comparing impatience in $(0,c_{¼}]$ versus $(c_{¾},52]$, testing $c_{¼}/2 - c_{⅛} = (c_{¾}+52)/2 - c_{⅞})$ two-sided. We found constant impatience rejected to the favor of increasing impatience (with < instead of =; $p = 0.001$).

TEST 9. For separability, we tested $s_{½} = c_{½}$, but rejected it ($p < 0.01$) to the favor of $s_{½} < c_{½}$.

TEST 10. For separability, we tested $s_{¾} = c_{¾}$, which was accepted ($p = 0.14$).

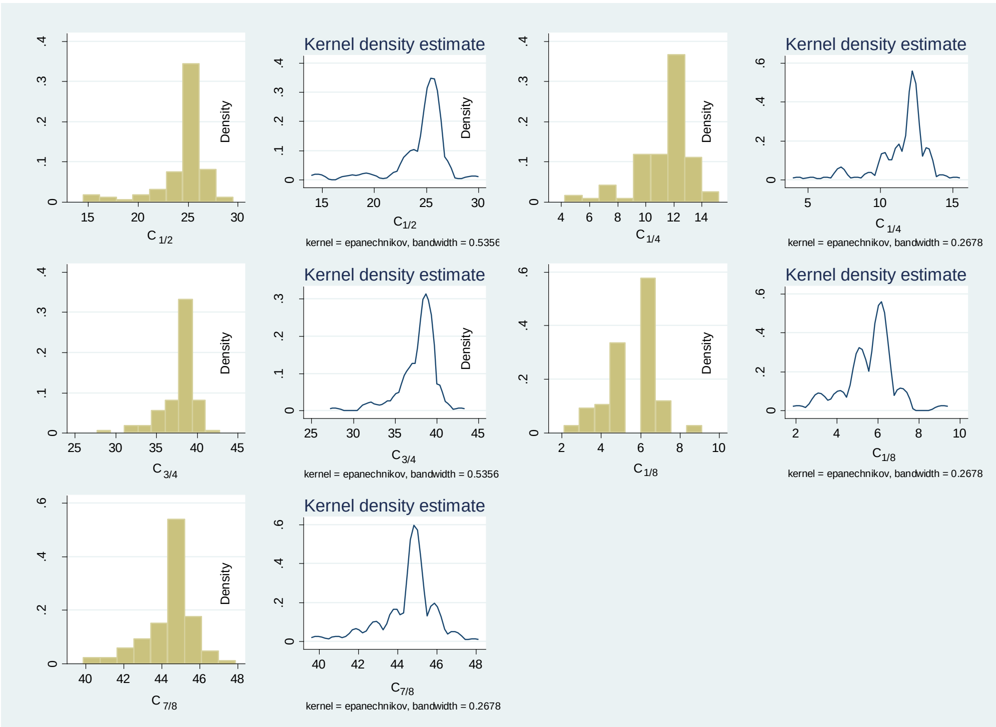
We next discuss an alternative rounding for testing qualitative preference conditions. There was a considerable group of subjects who had $u_{¼} = 13$, $u_{½} = 26$, and $u_{¾} = 39$, being 37 subjects. It suggests that many of these subjects are constant or very weak discounters, and our roundings may have been too much downward for them. Some hypotheses that we tested could have been favored or disfavored by the rounding chosen for these subjects. Hence we repeated all tests with the 60 subjects that remained after removing those 37 subjects. Removing subjects with constant discounting as done here should not affect the directional hypotheses tested. We found the same conclusions, with the same p-values, with the following exceptions. The main changes concern the tests of constant impatience. The decreasing impatience in test 7 ($c_{¾}-26 - c_{¼} > 0$) is no more significant ($p = 0.22$ two-sided), and neither is the increasing impatience in test 8 ($c_{¼}/2 - c_{⅛} > (c_{¾}+52)/2 - c_{⅞}$) ($p = 0.19$ two-sided). Some minor changes: Test 5 ($c_{¾} < (c_{½} + 52)/2$) now has $p = 0.003$; test 9 ($s_{½} = c_{½}$) now has $p = 0.017$; test 10 ($s_{¼} = c_{¼}$) now has $p = 0.81$.

WA3. Further results for the DM

The interval midpoints used in the graph of the discount factors derived from the cumulative discount weighting are 2.77, 8.51, 17.96, 31.12, 41.13, and 48.25 weeks, respectively. The corresponding discount factors for the vertical axis are 1, 0.94, 0.855, 0.835, 0.826, and 0.74.

For all c_p values, all the median values exceed the mean values, indicating negative skewness with the left tail longer than the right tail. It is confirmed by histogram and kernel density functions in Figure WA1.

FIGURE WA1. Histograms and kernel density curves for the c_p observations



Negative skewness is confirmed by the skewness/kurtosis tests for normality, with $p < 0.05$ for all kurtosis tests and four of the five skewness tests, and $p = 0.092$ for the remaining skewness test. Therefore, no c_p is normally distributed.

Skewness/Kurtosis tests for Normality_DM

Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	Joint Adj chi2(2)	Joint Prob>chi2
$\mathbf{c_{1/8}}$	97	0.0924	0.0370	6.66	0.0358
$\mathbf{c_{1/4}}$	97	0.0000	0.0008	28.22	0.0000
$\mathbf{c_{1/2}}$	97	0.0000	0.0000	39.42	0.0000
$\mathbf{c_{3/4}}$	97	0.0000	0.0000	33.28	0.0000
$\mathbf{c_{7/8}}$	97	0.0002	0.0126	16.21	0.0003

We next give the statistics showing that the difference between the area under the DM cumulative discount weight function and the area under the diagonal is positive, confirming impatience.

Signrank DM_area=0

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	81	4221	2327.5
negative	14	434	2327.5
zero	1	1	1
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties -1148.25

adjustment for zeros -0.25

adjusted variance 73735.50

Ho: DM_area = 0

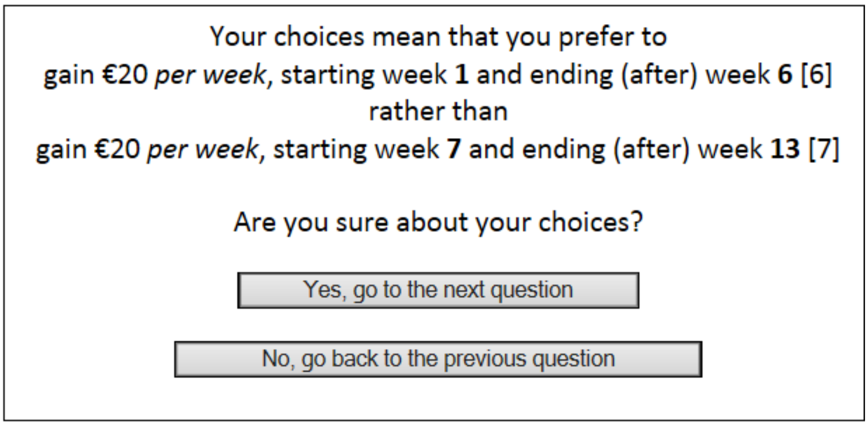
z = 6.973

Prob > z = 0.0000

WA3. Details of experiment

After subjects had completed a choice list, they clicked on a "submit my choices" button to go to the next page, which showed an implication of their choices (Figure WA2). For instance, if a subject chose as in Figure 5.1 in the main text, with indifference value 5.5, then after clicking the "submit my choices" button, the page shown in Figure WA.2 appeared. Subjects had to confirm the implied preferences to go to the next question. If they did not confirm, they went back to the previous page and filled out the choice list again.

FIGURE WA.2. Implication of the choice



Web Appendix WB: Further details for the UM

For each variable d_j^u, the median is larger than the mean, indicating negative skewness and failure of normal distribution. The following table confirms this using statistical tests, and rejecting normal distributions.

TABLE. Skewness/kurtosis tests rejecting normality of UM observations

Skewness/Kurtosis tests for Normality_UM					
				Joint	
Variable	Obs	Pr(Skewness)	Pr(Kurtosis)	Adj chi2(2)	Prob>chi2
$\mathbf{d_4^u}$	96	0	0.0004	33.81	0
$\mathbf{d_{12}^u}$	96	0	0.06	16.94	0.0002
$\mathbf{d_{20}^u}$	96	0.0001	0.13	14.23	0.0008
$\mathbf{d_{28}^u}$	96	0.01	0.14	7.72	0.02
$\mathbf{d_{36}^u}$	96	0.04	0.004	10.94	0.004
$\mathbf{d_{44}^u}$	96	0.046	0.002	11.45	0.003
$\mathbf{d_{52}^u}$	96	0.13	0.0001	14.72	0.0006

By impatience, switching values λ in $90_3 0 \sim \lambda_j 0$ should be increasing in duration j. 15 subjects violate this requirement at least once.

The weeks used in the graph of the discount factors are 3, 4, 12, 20, 28, 36, 44 and 48.25 weeks[1]. The corresponding discount factors for the vertical axis are 1, 0.935, 0.874, 0.845, 0.794, 0.773, 0.747 and 0.739. The latter value suggests an annual discount factor of 30%.

Wilcoxon signed-rank tests confirmed impatience (discount factors decreasing over time) by comparing each consecutive discount factors. Impatience is confirmed (p=0.0000 for all).

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[1] Although we have the discount factor d_{52}^u of 52 weeks, we do not show it in the Figure. Instead, for the end point of the UM, we used the last midpoint of the DM, to have direct comparisons between the UM and the DM.

Signrank d4=d12

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	52	3618	1855
negative	1	92	1855
zero	43	946	946
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties 0.00

adjustment for zeros -6858.50

adjusted variance 68025.50

Ho: d4 = d12

z = 6.760

Prob > z = 0.0000

Signrank d12=d20

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	44	3224	1690.5
negative	2	157	1690.5
zero	50	1275	1275
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties 0.00

adjustment for zeros -10731.25

adjusted variance 64152.75

Ho: d12 = d20

z = 6.054

Prob > z = 0.0000

Signrank d20=d28

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	57	3875	1957.5
negative	1	40	1957.5
zero	38	741	741
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties 0.00

adjustment for zeros -4754.75

adjusted variance 70129.25

Ho: d28 = d36

z = 7.241

Prob > z = 0.0000

Signrank d28=d36

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	40	2916	1715.5

negative	7	515	1715.5
zero	49	1225	1225
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties 0.00

adjustment for zeros -10106.25

adjusted variance 64777.75

Ho: d28 = d36

z = 4.717

Prob > z = 0.0000

Signrank d36=d44

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	47	3306	1855
negative	6	404	1855
zero	43	946	946
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties 0.00

adjustment for zeros -6858.50

adjusted variance 68025.50

Ho: d36 = d44

z = 5.563

Prob > z = 0.0000

Signrank d44=d52

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	37	2801	1612.5
negative	6	424	1612.5
zero	53	1431	1431
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties 0.00

adjustment for zeros -12759.75

adjusted variance 62124.25

Ho: d36 = d44

z = 4.768

Prob > z = 0.0000

We next give the statistics showing that the difference between the area under the UM cumulative discount weight function and the area under the diagonal is positive, confirming impatience.

Signrank UM_area=0

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	96	4656	2328
negative	0	0	2328
zero	0	0	0
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties -0.13

adjustment for zeros 0.00

adjusted variance 74883.88

Ho: UM_area = 0

z = 8.507

Prob > |z| = 0.0000

Web Appendix WC: Further details in comparing the DM and the UM

WC1. Regressions

To see how concavity of the cumulative discount weights is related to individual characteristics, we regress (1) DM_area, (2) UM_area and (3) difference between the two areas on risk preference parameters ($\alpha, \beta, 1-\eta$) and demographics (gender, age, nationality (Dutch/non-Dutch)). The following table gives the results.

TABLE. LS Regression of areas on risk preference parameters and demographics

	(1)	(2)	(3)
α	-0.63	0.24	-0.88
	(0.66)	(0.48)	(0.72)
Pessimism	0.13	-2.47 ***	2.60 *
	(1.22)	(0.88)	(1.33)
Concavity of Utility	-0.12	-1.21 ***	1.09 ***
	(0.28)	(0.21)	(0.31)
Nation	0.28	-0.15	0.43
(1: Dutch; 0: Other)	(0.38)	(0.28)	(0.42)
Gender	-0.16	-0.27	0.11
(1: Male; 0: Female)	(0.39)	(0.28)	(0.42)
Age	0.10	0.15 ***	-0.05
	(0.07)	(0.05)	(0.08)
Constant	-0.83	1.41	-2.23
	(1.99)	(1.44)	(2.17)
R^2	0.03	0.40	0.21
N	96	96	96

* 0.05; ** 0.01; *** 0.001

Column (1) shows that none of the parameters in risk preference or demographics have impact on DM time preference. Column (2) shows that UM time preference is related with pessimism in probability weighting and concavity of utility and also age.

Column (3) shows that the difference between the two areas is related to concavity of utility. There is no correlation between age and concavity of utility ($p = 0.16$), and also no correlation between gender and concavity of utility ($p = 0.46$).

WC2. Further details

The discount factors of 48.25 weeks in the DM and that in the UM suggest that the mean (median) of discount factors of 52 weeks (δ_{52}) is 0.756 (0.839) for the DM and 0.716 (0.748) for the UM. Wilcoxon test shows that subjects do not discount significantly differently in the DM and in the UM ($p = 0.8665$).

Signrank DM_d52=UM_d52

Wilcoxon signed-rank test

sign	obs	sum	expected
positive	46	2374	2328
negative	50	2282	2328
zero	0	0	0
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties 0.00

adjustment for zeros 0.00

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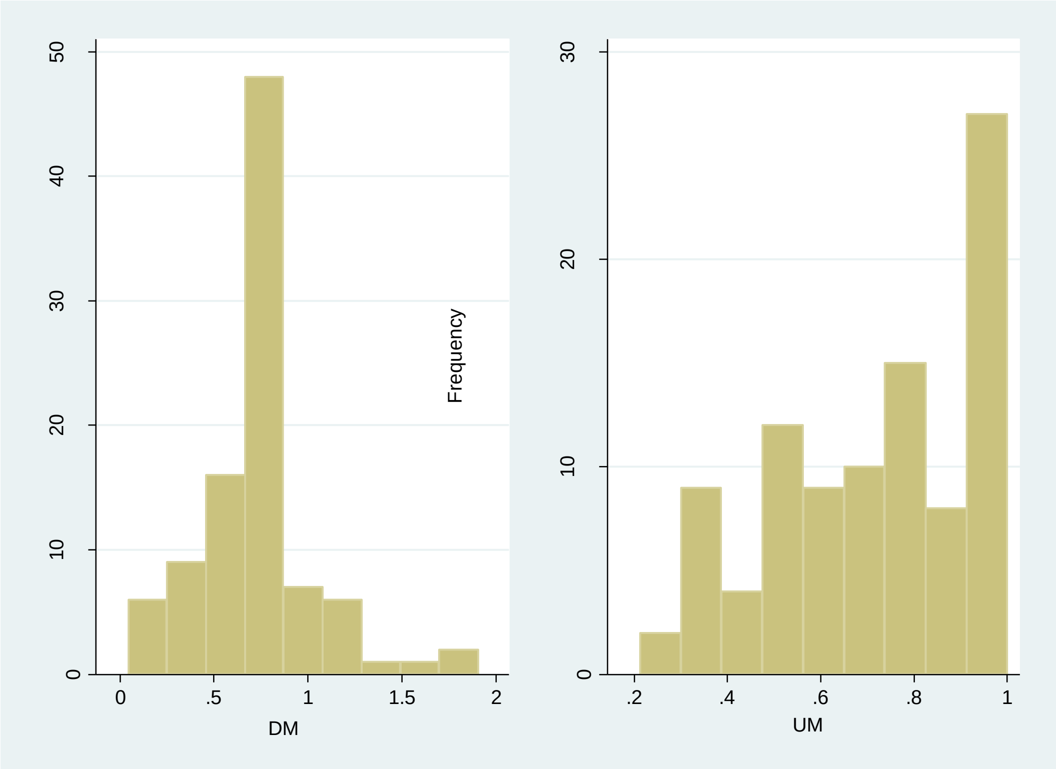
adjusted variance 74884.00

Ho: DM_d52=UM_d52

z = 0.168

Prob > z = 0. 8665

Histograms of discount factors are provided next.



We calculate annual discount rates r from $e^{-rt} = \delta_{52}$. The mean (median) of r is 0.409 (0.175) for the DM and 0.390 (0.291) for the UM. The difference is not significant ($p = 0.74$).

Signrank DM_r52=UM_r52

Wilcoxon signed-rank test

sign	obs	sum	expected
positive	50	2418	2328
negative	46	2238	2328
zero	0	0	0
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties 0.00

adjustment for zeros 0.00

adjusted variance 74884.00

Ho: DM_r52=UM_r52

z = 0.329

Prob > z = 0. 7422

DM_area and UM_area are the difference between area under the DM/UM cumulative discount weighting functions and area under the diagonal. Here is the output of the test showing that the UM area exceeds the DM area.

signrank UM_area=DM_area

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	55	2980	2328
negative	41	1676	2328
zero	0	0	0
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties 0.00

adjustment for zeros 0.00

adjusted variance 74884.00

Ho: UM_area = DM_area

z = 2.383

Prob > z = 0.0172

Here is the output of the test showing that the power function fitted to the DM area has a power different than 1, confirming concavity of C.

signrank DM_power=1

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	10	307	2328
negative	86	4349	2328
zero	0	0	0
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties -749.50

adjustment for zeros 0.00

adjusted variance 74134.500

Ho: DM_power = 1

z = -7.423

Prob > z = 0.0000

Here is the output of the test showing that the power function fitted to the UM area has a power different than 1, confirming concavity of C^u.

signrank UM_power=1

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	0	0	2328
negative	96	4656	2328
zero	0	0	0
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties -0.13

adjustment for zeros 0.00

adjusted variance 74883.88

Ho: UM_power = 1

z = -8.507

Prob > z = 0.0000

Here is the output of the test showing that the power function fitted to the UM area is not significantly different from that fitted to the DM area.

signrank UM_power=DM_power

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	50	2140	2328
negative	46	2516	2328
zero	0	0	0
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties 0.00

adjustment for zeros 0.00

adjusted variance 74884.00

Ho: UM_power = DM_power

z = -0.687

Prob > z = 0.4921

The following table shows the correlation between discount rate r, DM_area/UM_area and concavity of C.

	r(DM)	area(DM)	concavity(DM)	r(UM)	area(UM)	concavity(UM)
r(DM)	-----	P=0.000 ρ = 0.958	P=0.000 ρ= -0.898	P=0.008 ρ= 0.270	P=0.010 ρ=0.262	P=0.035 ρ =-0.215
area(DM)	-----	-----	P=0.000 ρ = -0.901	P=0.015 ρ=0.248	P=0.020 ρ=0.237	P=0.046 ρ=-0.205
concavity(DM)	-----	-----	-----	P=0.007 ρ=-0.342	P=0.001 ρ=-0.325	P=0.004 ρ=0.289
r(UM)	-----	-----	-----	-----	P=0.000 ρ = 0.989	P=0.000 ρ = -0.963
area(UM)	-----	-----	-----	-----	-----	P=0.000 ρ = -0.946
concavity(UM)	-----	-----	-----	-----	-----	-----

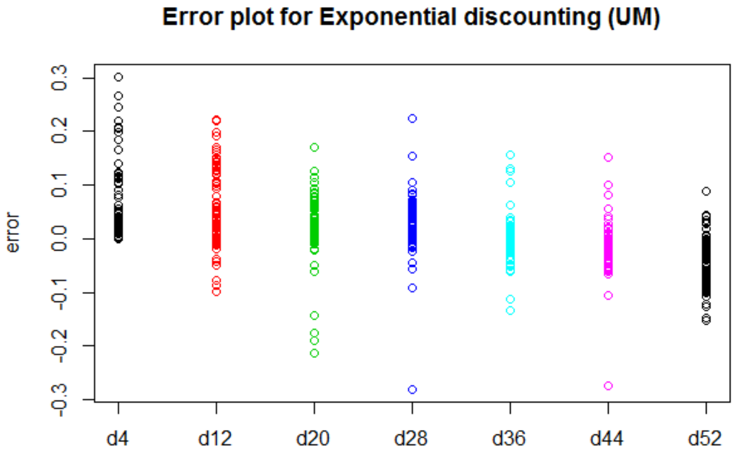
Web Appendix WD: Statistics for parametric fittings

The following table gives descriptives of parametric fittings of discounting on the individual level.

variable	mean	min	median	max	sd
r (exponential; UM)	0.009	0.000	0.006	0.046	0.009
α (hyperbolic; UM)	1.876	0.000	1.214	8.222	2.139
β (hyperbolic; UM)	0.249	0.000	0.055	3.981	0.557
d (unit invariance; UM)	0.576	-9.401	0.938	3.719	1.352
r (unit invariance; UM)	2.190	0.000	0.692	18.193	3.411
r (exponential; DM)	0.005	-0.012	0.002	0.037	0.008
α (hyperbolic; DM)	1.663	0.000	1.297	9.770	2.497
β (hyperbolic; DM)	0.139	-0.012	0.049	1.650	0.261
d (unit invariance; DM)	0.801	-1.281	0.893	1.954	0.490
r (unit invariance; DM)	1.858	0.000	0.246	19.982	2.970

The following shows tests for heteroskedasticity. In our paper, we fit three models for both UM data and DM data.

We first present the error term of the exponential model for UM data in the following figure.



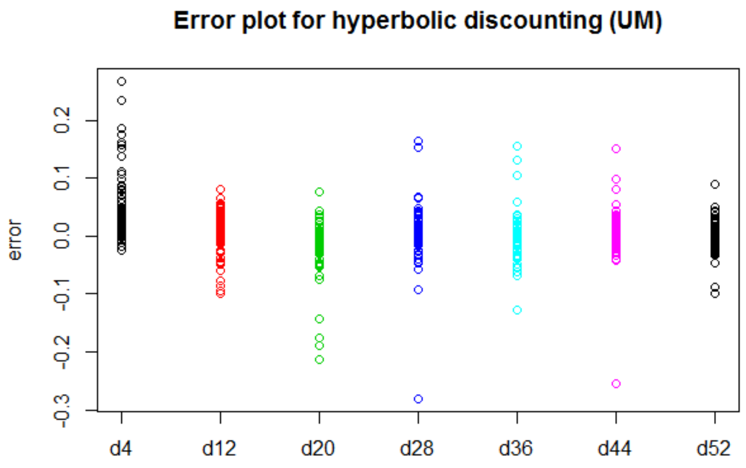
Levene's test of equality of variances rejects the null hypothesis that absolute deviations from the medians are the same across time ($p<0.01$).

modified robust Brown-Forsythe Levene-type test based on the absolute deviations from the median

data: error_exponential_UM

Test Statistic = 5.2847, p-value = 2.46e-05

In the following, we present the error term of the hyperbolic model for UM data.



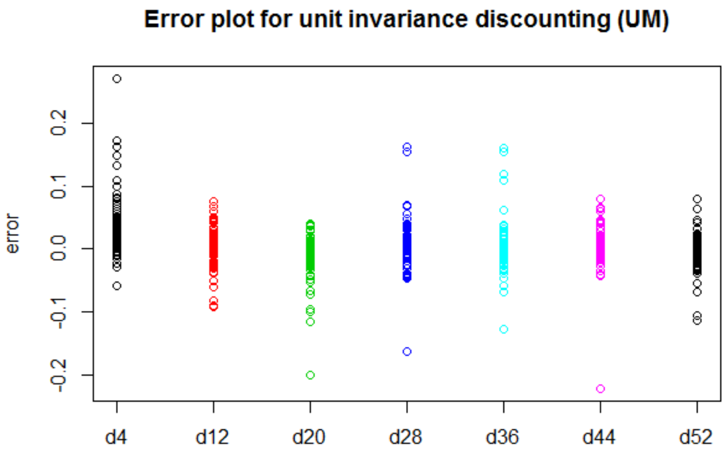
For the hyperbolic discounting model, Levene's test cannot reject the null that absolute deviations from the medians are the same across time (p>0.05).

modified robust Brown-Forsythe Levene-type test based on the absolute deviations from the median

data: error_hyperbolic_UM

Test Statistic = 2.0049, p-value = 0.06292

The following figure shows the error term of the unit invariance model for UM data.



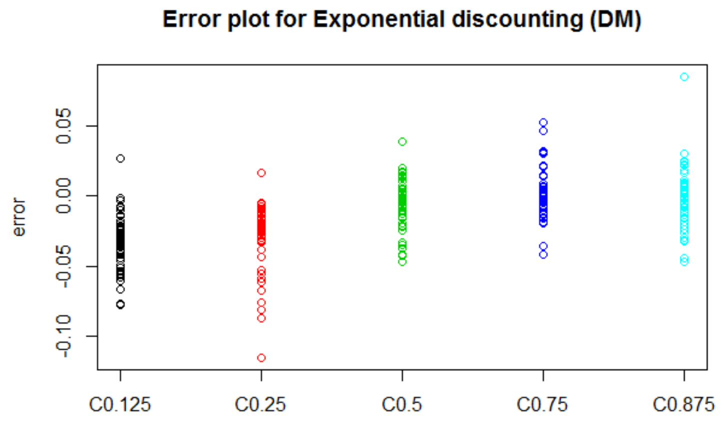
Levene’s test cannot reject the null that absolute deviations from the medians are the same (p>0.05).

modified robust Brown-Forsythe Levene-type test based on the absolute deviations from the median

data: error_unit.invariance_UM

Test Statistic = 1.9809, p-value = 0.06621

The following figure shows the error plot for the exponential discounting model for DM data.



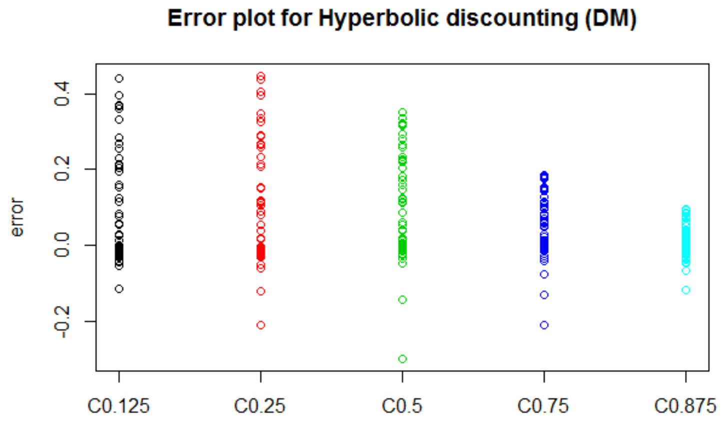
Levene's test cannot reject the null. So deviations from the medians are constant ($p>0.05$).

modified robust Brown-Forsythe Levene-type test based on the absolute deviations from the median

data: error_exponential_DM

Test Statistic = 1.6655, p-value = 0.1568

In the following figure, we show the error plot for the hyperbolic discounting model.



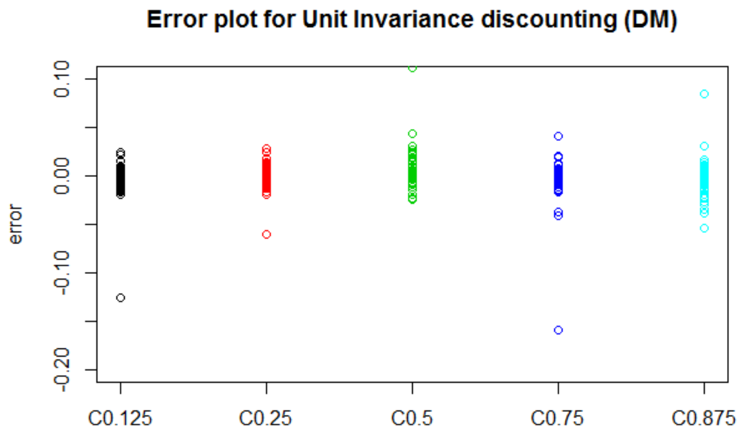
It can be spotted from visual inspection that the error terms in each column have different variances. Levene's test confirmed this (p<0.01).

modified robust Brown-Forsythe Levene-type test based on the absolute deviations from the median

data: error_hyperbolic_DM

Test Statistic = 104.5901, p-value < 2.2e-16

The following figure gives the error plot for the unit invariance discounting model.



Leneve's test cannot reject the null that the variances are the same (p>0.1).

modified robust Brown-Forsythe Levene-type test based on the absolute deviations from the median

data: error_unit.invariance_DM

Test Statistic = 1.6136, p-value = 0.1696

The following output shows that the discount rate r of exponential discounting of the UM exceeds that of the DM.

signrank r (exponential, UM)= r (exponential, DM)

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	60	3344	2328
negative	36	1312	2328
zero	0	0	0
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties 0.00

adjustment for zeros 0.00

adjusted variance 74884.00

Ho: r (exponential, UM)= r (exponential, DM)

z = 3.713

Prob > z = 0.0002

The following output shows that the α parameter of hyperbolic discounting of the DM is not significantly different from that of the UM.

Signrank α (hyperbolic, DM) = α (hyperbolic, UM)

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	40	1998	2328
negative	56	2658	2328
zero	0	0	0
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties 0.00

adjustment for zeros 0.00

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adjusted variance 74884.00

Ho: α (hyperbolic, DM)= α (hyperbolic, UM)

z = -1.206

Prob > z = 0.2278

The following output shows that the β parameter of hyperbolic discounting of the UM is not significantly different from that of the DM.

Signrank β (hyperbolic, DM) = β (hyperbolic, UM)

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	41	2049	2328
negative	55	2607	2328
zero	0	0	0
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties 0.00

adjustment for zeros 0.00

adjusted variance 74884.00

Ho: β (hyperbolic, DM) = β (hyperbolic, UM)

z = -1.020

Prob > z = 0.3079

The following output shows that the d parameter of unit invariance of the DM exceeds that of the UM, but not significant.

Signrank d (unit invariance, UM) = d (unit invariance, DM)

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	45	2026	2328
negative	51	2630	2328
zero	0	0	0
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties 0.00

adjustment for zeros 0.00

adjusted variance 74884.00

Ho: d (unit invariance, UM) = d (unit invariance, DM)

z = -1.104

Prob > z = 0.2698

The following output shows that the r parameter of unit invariance of the DM is not significantly different from that of the UM.

Signrank r (unit invariance, UM) = r (unit invariance, DM)

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	48	2424	2328
negative	48	2232	2328
zero	0	0	0
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties -1.25

adjustment for zeros 0.00

adjusted variance 74882.75

Ho: r (unit invariance, UM) = r (unit invariance, DM)

z = 0.351

Prob > z = 0.7257

The following output gives descriptive statistics of the Akaike information criterion (AIC).

variable	mean	min	median	max	sd
exponential _UM	-3.026	-6.708	-3.261	0.452	1.229
exponential _DM	-3.559	-3.990	-3.676	-2.147	0.475
hyperbolic _UM	-1.552	-4.916	-1.684	1.943	1.025
hyperbolic_ DM	-1.659	-2.184	-1.799	-0.017	0.440
unit invariance_ UM	-1.628	-5.084	-1.699	1.931	0.953
unit invariance_ DM	-1.647	-2.269	-1.779	-0.017	0.462

The following output shows that AIC for exponential discounting of the UM exceeds that of the DM, implying that the DM has a better fit.

Signrank AIC (exponential, UM) = AIC (exponential, DM)

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	72	3577	2328
negative	24	1079	2328
zero	0	0	0
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties 0.00

adjustment for zeros 0.00

adjusted variance 74884.00

Ho: AIC (exponential, UM) = AIC (exponential, DM)

z = 4.564

Prob > z = 0.0000

The following output shows that AIC for hyperbolic discounting of the UM, while exceeding that of the DM somewhat (suggesting that the DM has a better fit), does not do so significantly.

Signrank AIC (hyperbolic, UM) = AIC (hyperbolic, DM)

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	53	2753	2328
negative	43	1903	2328
zero	0	0	0
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties 0.00

adjustment for zeros 0.00

adjusted variance 74884.00

Ho: AIC (hyperbolic, UM) = AIC (hyperbolic, DM)

z = 1.553

Prob > z = 0.1204

The following output shows that AIC for unit invariance discounting of the DM and the UM do not differ significantly.

Signrank AIC (unit.invariance, UM) = AIC (unit.invariance, DM)

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	52	2582	2328
negative	44	2074	2328
zero	0	0	0
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties 0.00

adjustment for zeros 0.00

adjusted variance 74884.00

Ho: AIC (unit.invariance, UM) = AIC (unit.invariance, DM)

z = 0.928

Prob > z = 0.3533

The following output shows that for the UM the AIC for unit invariance discounting exceeds that of exponential discounting (so that exponential has a better fit).

Signrank AIC (unit.invariance, UM) = AIC (exponential, UM)

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	95	4639	2328
negative	1	17	2328
zero	0	0	0
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties -0.13

adjustment for zeros 0.00

adjusted variance 74883.88

Ho: AIC (unit.invariance, UM) = AIC (exponential, UM)

z = 8.445

Prob > z = 0.0000

The following output shows for the UM that the AIC for hyperbolic discounting exceeds that of unit invariance (so that unit invariance has a better fit).

Signrank AIC (unit.invariance, UM) = AIC (hyperbolic, UM)

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	17	634	2328
negative	79	4019	2328
zero	0	0	0
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties -0.13

adjustment for zeros 0.00

adjusted variance 74883.88

Ho: AIC (unit.invariance, UM) = AIC (hyperbolic, UM)

z = -6.179

Prob > z = 0.0000

The following output shows for the UM that the AIC for hyperbolic discounting exceeds that of exponential discounting (so that exponential discounting has a better fit).

Signrank AIC (hyperbolic, UM) = AIC (exponential, UM)

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	88	4394	2328
negative	8	262	2328
zero	0	0	0
all	96	4656	4656

unadjusted variance 68114.75

adjustment for ties 0.00

adjustment for zeros 0.00

adjusted variance 68114.75

Ho: AIC (hyperbolic, UM) = AIC (exponential, UM)

z = 7.370

Prob > z = 0.0000

The following output shows for the DM that the AIC for hyperbolic discounting exceeds that of unit invariance (so that unit invariance has a better fit), but not significantly.

Signrank AIC (unit.invariance, DM) = AIC (hyperbolic, DM)

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	31	1995	2326.5
negative	63	2658	2326.5
zero	2	3	3
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties -370.75

adjustment for zeros -1.25

adjusted variance 74512.00

Ho: AIC (unit.invariance, DM) = AIC (hyperbolic, UM)

z = -1.214

Prob > z = 0.2246

The following output shows for the DM that the AIC for hyperbolic discounting exceeds that of exponential discounting (so that exponential discounting has a better fit).

Signrank AIC (exponential, DM) = AIC (hyperbolic, DM)

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	0	0	2328
negative	96	4656	2328
zero	0	0	0
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties -771.00

adjustment for zeros 0.00

adjusted variance 74113.00

Ho: AIC (exponential, DM) = AIC (hyperbolic, DM)

z = -8.551

Prob > z = 0.0000

The following output shows for the DM that the AIC for unit invariance exceeds that of exponential discounting (so that exponential discounting has a better fit).

Signrank AIC (exponential, DM) = AIC (unit.invariance, DM)

Wilcoxon signed-rank test

sign	obs	sum ranks	expected
positive	0	0	2328
negative	96	4656	2328
zero	0	0	0
all	96	4656	4656

unadjusted variance 74884.00

adjustment for ties -370.75

adjustment for zeros 0.00

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adjusted variance 74513.25

Ho: AIC (exponential, DM) = AIC (unit.invariance, DM)

z = -8.528

Prob > z = 0.0000

Web Appendix WE: Theoretical possibility to manipulate in the adaptive experiment

As explained in the main text, the possibility for subjects to manipulate in the experiment is only a theoretical problem because in reality it is impossible for subjects to see through the design without knowing it beforehand. Even readers who have studied the design will need considerable time before even being able to specify how to benefit from manipulation. We now consider the theoretical case where someone knows the entire design and has used considerable time to think about manipulations, which is our case as authors of this paper. We assume that all answers are equally likely to be implemented for real, and that the prize to be won is fixed.

A wrong answer in the measurement of $c_{½}$ will bring no net gain in the measurements of $c_{¼}$ and $c_{¾}$ because the time period gained for one of these two is the time period lost for the other. It will neither bring net gains in the measurements of $c_{⅛}$, and $c_{⅞}$ because, again, the duration gained for one is the duration lost for the other. The only benefit possible is from making $c_{¼}$ too large (or, similarly, making $c_{¾}$ too small). Then in some followup questions for $c_{⅛}$ there is a gain, always less than half the loss suffered due to the preceding wrong answer (but in some there is a loss). But there are more, usually around 12, choice questions in the choice list for $c_{⅛}$. Hence in expectation one gains about $p \times (8/2-1)$ times the error made, where p is the probability of the question being selected for real. Given that $p \approx 0.01$, this is a moderate gain.